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Estudo inicial da aplicação de Redes Neurais Artificiais para a previsão de padrões de fluxo em escoamento multifásico

Initial study of application of Neural Networks to predict flow patterns in multiphase flow

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O estudo dos padrões de fluxo no escoamento multifásico é bastante usado na indústria do petróleo. Muitas ferramentas estão sendo desenvolvidas para auxiliar na previsão desses padrões de fluxo. A Inteligência Artificial é uma ferramenta com alto desempenho e grande uso para caracterizar fenômenos físicos, podendo ser usada para a previsão desses padrões de fluxo no escoamento multifásico. Desenvolvemos uma Rede Neural Artificial para reconhecer e prever os padrões de fluxo do escoamento multifásico utilizando uma base de dados experimentais. Além do uso da rede neural também foi realizado o tratamento dos dados para melhorar a eficácia da ferramenta. Para uma análise completa, treinamos a rede com diversas combinações dos padrões, sendo os padrões: intermitente, anular e estratificado. Não utilizamos o padrão de bolhas devido a diferença da quantidade de dados disponíveis entre esse e os demais padrões. A rede apresentou uma eficácia de até 90% em algumas combinações da análise entre os padrões de fluxo. Analisando os 3 padrões juntos, a rede apresentou uma eficácia de aproximadamente 67%. Analisamos os resultados do comportamento do erro da rede durante o seu treinamento e a taxa de acerto após o treinamento. Nesse artigo utilizamos a rede neural artificial como ferramenta para prever o padrão de fluxo no escoamento multifásico, com o máximo de parâmetros do escoamento disponível.
Palavras-chave: escoamento multifásico, padrão de fluxo, redes neurais.

The study of flow patterns in multiphase flow is widely used in the oil industry. Many tools are being developed to help predict these flow patterns. Artificial Intelligence is a tool with high performance and great use to characterize physical phenomena and can be used to predict these flow patterns in multiphase flow. We developed an Artificial Neural Network to recognize and predict the flow patterns of multiphase flow using an experimental database. In addition to using the neural network, data processing was also carried out to improve the tool's effectiveness. For a complete analysis, we trained the network with several combinations of the patterns, the patterns being: intermittent, annular and stratified. We did not use the bubble pattern due to the difference in the amount of data available between this and the other patterns. The net was up to 90% effective in some combinations of the analysis between flow patterns. Analyzing the 3 patterns together, the network showed an efficiency of approximately 67%. We analyzed the results of the net error behavior during its training and the hit rate after training. In this article, we used the artificial neural network as a tool to predict the flow pattern in the multiphase flow, with the maximum available flow parameters.
Keywords: multiphase flow, flow pattern, neural network.

1. INTRODUCTION

Multiphase flow is the flow of a mixture with more than one phase, whether liquid, gas or solid [1]. In the study of fluid flow, as in some areas of the oil industry, for example, the behavior of the flow with the gaseous and liquid phases is studied. When there are these two phases in flow, depending on the velocity and other fluid and flow parameters, the flow can take on a specific flow pattern. The prediction of these flow patterns, considering liquid and gas in tubes, is an important problem for the study of two-phase flow [2].

The flow pattern depends on your direction, fluid properties such as density and viscosity, pressure and many other parameters. Briefly, we could classify the flow patterns into bubble, annular, stratified and intermittent patterns also known as slug. The bubble pattern is characterized by the flow with bubbles dispersed in the liquid medium. The stratified pattern has the two phases separated generally at low speeds. The annular pattern, with slightly higher velocities, generates a gas flow with a liquid film on the tube walls. The intermittent pattern is characterized by the alternating flow of the liquid phase and the gas phase [3].

The study of flow patterns is important in the oil industry to understand and predict the behavior of fluids during flow. Studying pattern transitions is a great challenge. Taitel and Dukler (1976) [4] prepare a model for predicting the transition of flow patterns with an analytical approach. Unified models are developed to predict the transition of flow patterns beyond pressure gradient and other flow characteristics. Hong-Quan, Wang and Sarica (2003) bases its model on the dynamics of the slug pattern because its transition limits are shared with the other patterns, in addition to considering the flow angle from -90° to 90° [5]. After several studies of unified models with different inclinations, there is a proposal for a unified model that covers all inclination angles [2].

In the oil field, machine learning has been widely used in research due to its great potential to solve problems [6-10]. This deep learning method has a great performance in price prediction, in which there is a study in the oil price forecast with this tool [11]. In the investigation of flow patterns, there are several studies with neural networks. Al Naser et al. (2016) [12] developed an artificial neural network in Matlab to study different cases of flow patterns, showing the effectiveness of this tool with 97% accuracy in prediction, highlighting the importance of data pre-processing. Bahrami et al. (2019) [13] uses neural networks to determine three-phase, water-oil-gas flow rates, validated with field data. Artificial neural networks are a subcategory of machine learning, but with deeper learning, where he uses an analogy of the functioning of the human brain for his learning process [14].

Our objective is to evaluate initial studies on artificial neural networks to help identify the flow patterns of the multiphase flow. We analyzed its performance and efficiency and its relation to the flow phenomenon, studying how the network behaves with the greatest amount of input variables and its ability to represent the environment in which it is inserted.

2. MATERIAL AND METHODS

The structure of the methodology of our work follows in accordance with Figure 1. Firstly, we need a database to perform the training of the artificial neural network. The pre-Processing these data is of fundamental importance for efficient training after the processing the data, we perform learning of the artificial neural network. Finally, we evaluated the network performance. The percentage of data related to training and testing is 80%/20%.

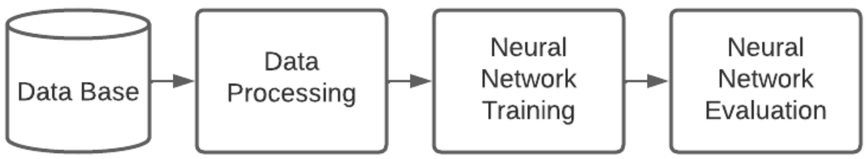

Figure 1- Methodology structure.

Problem data is needed to train and test our neural network. Specifically, we need the physical parameter data for the input and the patterns of corresponding flow to the outlet. The data obtained were taken from the study by Pereyra et al. (2012) [15], where a database of flow patterns from 12 experimental studies was collected.

Data handling is essential for network efficiency. We use the Python language for data processing, removing examples with missing data, organizing in the same number of examples for all patterns, applying the natural log to the data and scrambling the data to be presented to the network. The organization of the same number of examples for the patterns is necessary so that the network does not learn more about a specific pattern than the other. After filtering and removing missing data, the number of examples for the pattern of bubbles was smaller in relation to the other patterns, therefore, this pattern was not selected for the neural network developed in this study.

By scrambling the data to be presented to the network, we improve the learning from the network and prevent it from specializing in just one pattern in the process of learning. According to Al Naser et al. (2016) [12] the use of pre-processing by applying a normalization with natural logarithmic on input data improves neural network performance and reduces overlap between flow patterns.

The neural network processing is based on the artificial neuron, it consists of a fundamental information processing unit for a neural network [14]. The components of an artificial neuron are: input signals, synaptic weights, activation function and output. These components are illustrated in Figure 2. An input (x_j) is multiplied by a weight (w_{kj}) of neuron k. After this process of each input with its synapse, the adder will perform a linear combination. The result of the adder will go through a defined activation function, which will restrict the output amplitude, generating the output of neuron k.

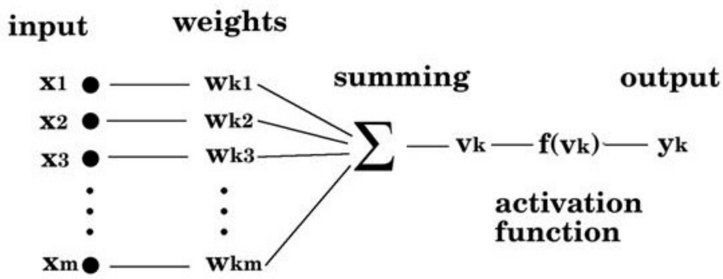

Figure 2 - Simplified diagram of an artificial neuron.

Among the different types of neural networks, we used the multilayer perceptron, due to its high performance in solving problems. A layer is composed of several artificial neurons performing the processing described above. Neural networks can contain several hidden layers. The multilayer perceptron is composed of the input layer, hidden layer, and output layer. The input layer is where data is entered into the neural network. In the hidden layer, the processing with the inputs is carried out. The output layer provides the output from the network. Figure 3 illustrates the structure of a multilayer perceptron, highlighting its layers.

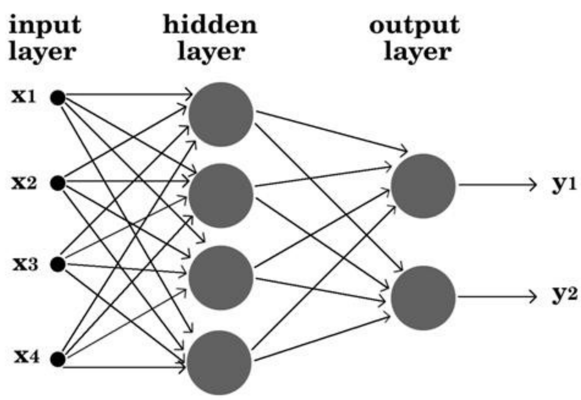

Figure 3 - Simplified diagram of a neural network.

The learning process of the neural network is the main ability for it to represent the system in which it is exposed. Briefly, the network learns from its synaptic weight changes. According to Mendel and McLaren (1970) [16], learning the neural network is a process in which its parameters are adapted according to the environment in which it is inserted.

The network can be supervised or unsupervised, whether there is a desired response. Our network is based on supervised learning, that is, the examples that are presented for the network contain the input and the desired answer for that input. The network does not know the desired response, it receives the input and generates its output, where we compared it with the desired response, generating an error, this is how supervised learning works. We used the error backpropagation algorithm for the neural network learning process. The network goes through the propagation from input to output, going through the processes explained above, where it generates an error, this error returns through the network correcting the synaptic weights, hence the name of the algorithm.

Problem data is needed to train and test our neural network. Specifically, we need the physical parameter data for the input and the corresponding flow patterns for the output. The data obtained were taken from the study by Pereyra et al. (2012) [15], where a flow pattern database of 12 experimental studies was collected.

Data handling is essential for network efficiency. We used the Python language for data processing, removing examples with missing data, organizing the same number of examples for all patterns, applying the natural log to the data and shuffling the data to be presented to the network. The organization of the same number of examples for the patterns is necessary so that the network does not learn more about a specific pattern than another. After filtering and removing missing data, the number of examples for the bubble pattern was smaller compared to the other patterns, so this pattern was not selected for the neural network developed in this study.

By scrambling the data to be presented to the network, we improved the network's learning curve and prevent it from specializing in just one pattern in the learning process. According to Al Naser et al. (2016) [12] the use of pre-processing by applying a normalization with the natural logarithmic to the input data improves the performance of the neural network and reduces the overlap between flow patterns.

3. RESULTS

We used as input parameters of the neural network, pressure, liquid density, gas density, liquid viscosity, gas viscosity, tube diameter, tube angle, liquid surface velocity and gas surface velocity. The network we used is fully wired, that is, all inputs go through all hidden-layer neurons and all hidden-layer outputs are inputs from the output neurons. The activation function used in the network is the sigmoid function. Analyzes are made for 2 patterns and 3 patterns, changing the number of neurons in the hidden layer.

Neural network training with 2 patterns had a low time. Using 2400 examples for training with 9 input parameters in each example. The training time is based on the variation of the number of neurons in the hidden layer and the number of epochs defined as stopping criteria. The hidden layer neurons range from 25, 50, 75 and 100 neurons. The criterion for stopping by epochs varies from 100, 500, 1000, 2000, 5000 and 10000 epochs. The behavior of the training time shows a linear relationship with the number of epochs in each configuration of neurons in the hidden layer. For the configuration of 25 neurons with the 100 epoch stopping criterion, the training time was 2.655 seconds. The longest training time was for the configuration of 100 neurons and 10000 epochs, with a time of approximately 17 minutes.

The output layer of the neural network for predicting 2 patterns has 2 output neurons. Each example returns output values (y_k) for each output neuron, generating an error (e_k), compared to the desired response (d_n). After training all the examples, that is, one epoch, all the mean squared errors of the training examples are added, according to equation 1. This sum is analyzed for each epoch until the end of the training, 10000 epochs, where we could observe the error behavior during training and compare it with the performance of the network when performing the test.

$$E = \sum_{n=1}^{2400} \frac{1}{2} \sum_{k=1}^{2} e_k^2 \quad (1)$$

The values of the sum of the root mean square error by epoch are analyzed for 25, 50, 75 and 100 neurons in the hidden layer. For each configuration in the hidden layer the network behaves differently. The analyzes are performed for each possible configuration with 2 patterns, and the data obtained are from the Intermittent, Stratified and Annular patterns.

3.1 Case 1: Stratified and Annular

During the training of the artificial neural network, all the examples for the network are presented. We calculated the sum of the root mean square error of each example at the end of each epoch. During the entire training we performed the error analysis for each configuration of neurons in the hidden layer. Figure 4 shows the behavior of the network error during training.

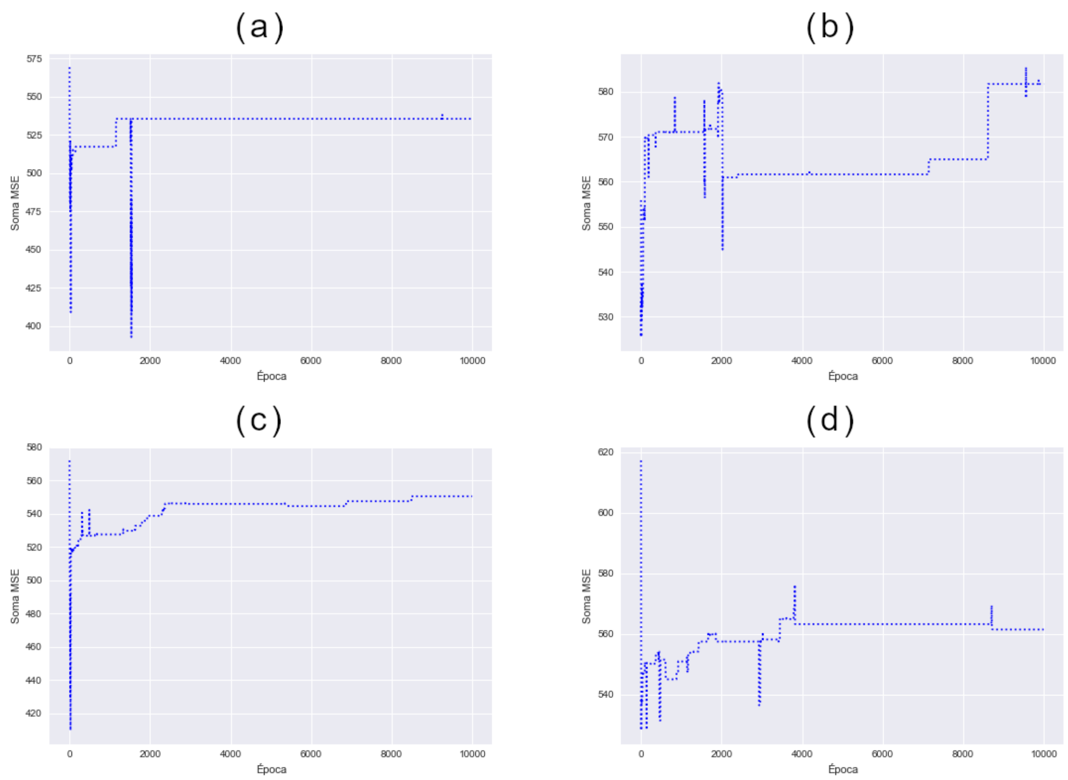

Figure 4 - Sum of the root mean square error of the neural network for Stratified and Annular with 25 neurons in hidden layer (a), 50 neurons in hidden layer (b), 75 neurons in hidden layer (c) and 100 neurons in hidden layer (d).

Figure 4a shows the behavior of small drops, close to 30 units, in the values of the sum of the mean squared error and then remains constant. Figure 4b shows an increase in the values of the sum of the mean square error with increasing epochs. Figure 4c resembles the behavior of Figure 4a. Finally, Figure 4d has an abrupt drop at the beginning of the simulation and then becomes stable.

The net is tested after your training. Testing is performed with new data that was not presented to the network during training. The network hit rate for the new data is calculated and analyzed which configuration presented the best performance.

Figure 5 shows the hit rate for each of the neuron configurations in the hidden layer, being 25, 50, 75 and 100 according to the number of epochs, 100, 500, 1000, 2000, 5000 and 10000.

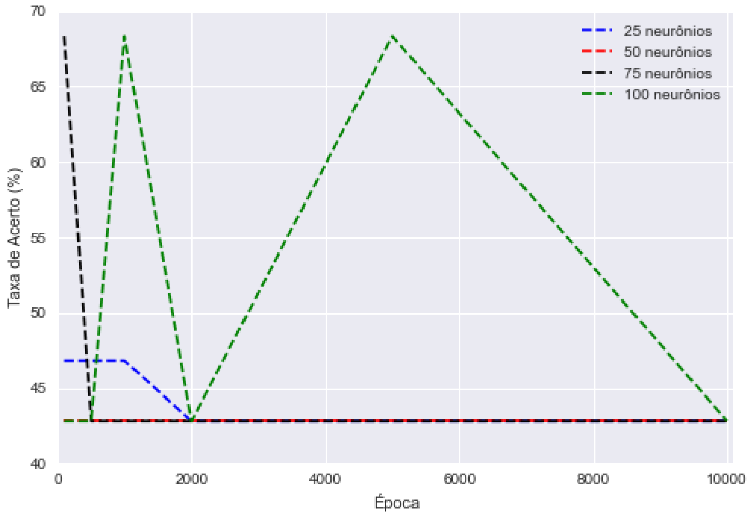

Figure 5 - Stratified and Annular Standards Hit Rate.

It is observed that the highest hit rate values are 68% for 100 neurons in the hidden layer in 1000 and 5000 epochs, also for 75 neurons in the hidden layer for 100 epochs. Analyzing the behavior of Figure 4d, the decay of the sum of the mean square error and right after its constancy was the one with the highest efficiency in the tests. The correctness for the Stratified pattern was higher compared to the Annular one, with 80% correctness of the Stratified data and 52% of the Annular data.

3.2 Case 2: Intermittent and Annular

We performed the training of the neural network with the Intermittent and Annular patterns, where the sum mean squared error of each example is calculated and its values are added at the end of each epoch. The error behavior is analyzed in Figure 6, for each configuration of neurons in the hidden layer.

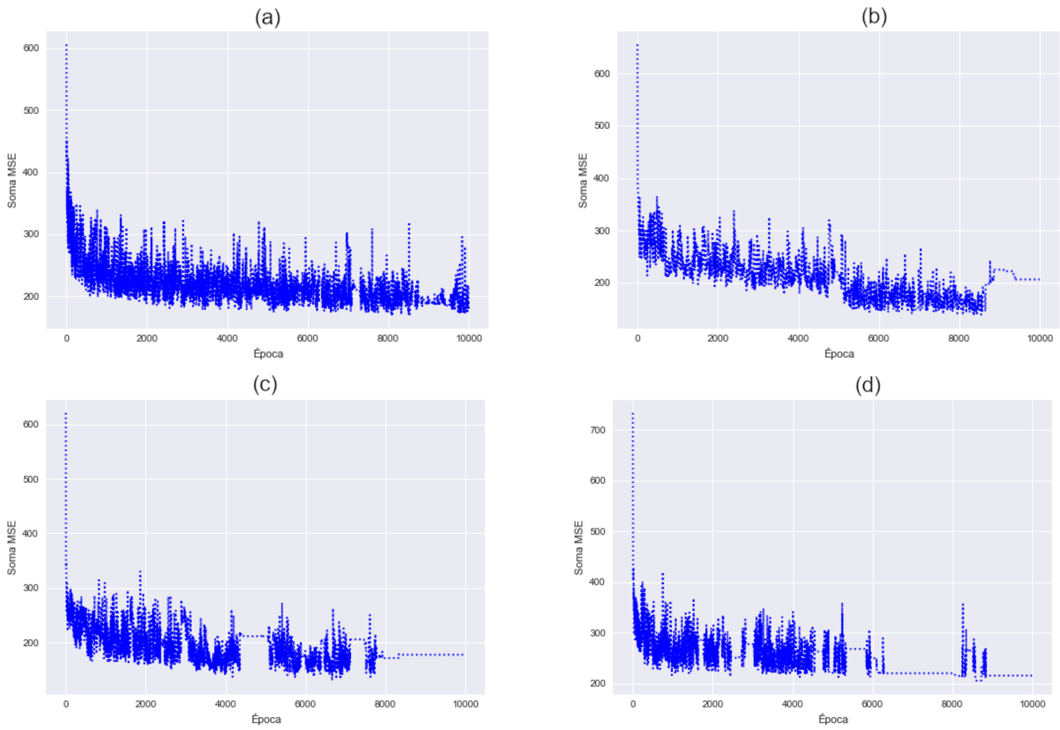

Figure 6 - Sum of the root mean square error of the neural network for Intermittent and Annular with 25 neurons in hidden layer (a), 50 neurons in hidden layer (b), 75 neurons in hidden layer (c) and 100 neurons in hidden layer (d).

All configurations of the neural network in Figure 6 exhibit decay behavior. Some distinct points tend to lose the oscillation present in the decay, such as the region between 6000 and 8000 epochs in Figure 6d. These regions cannot be examined due to the tests being performed at predetermined points of epochs.

Tests are performed after the neural network training phase, with new data that has not yet been presented during training. It is observed in the tests that the majority had an excellent percentage rate of correct answers, Figure 7.

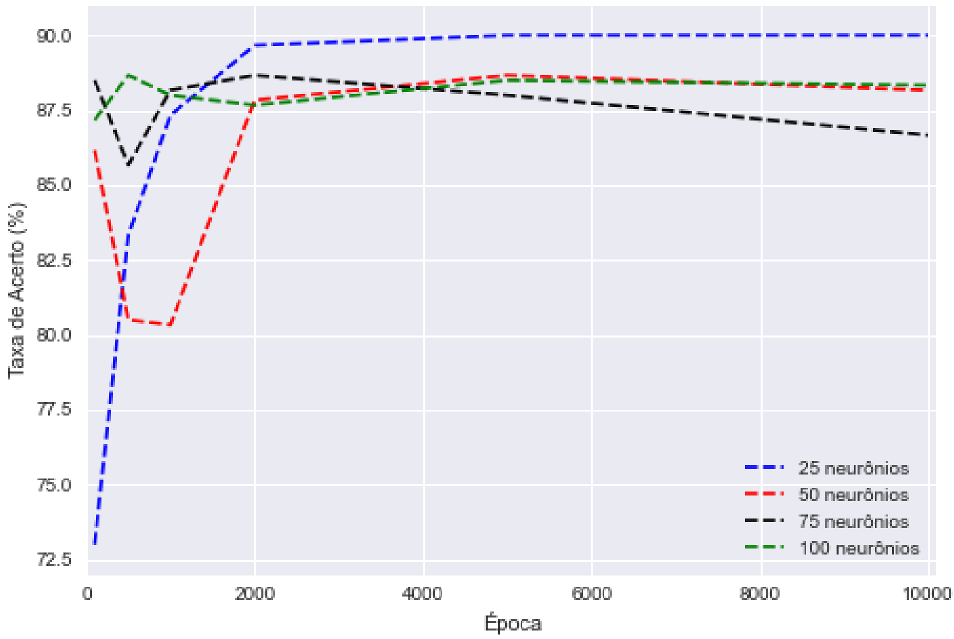

Figure 7 - Intermittent and Annular pattern hit rate.

Most of the tests showed between 80% and 90% correctness, as shown in Figure 7, they are great values for the neural network, showing that there was a good classification between the Intermittent pattern and the Annular pattern. Both patterns have a high hit rate, for the Intermittent pattern there was 96% hits of the total data of this pattern, for the Annular pattern, there was 82% hits only for this pattern. The hit rate for 50 neurons in the hidden layer (red) close to 1000 epochs has a non-uniform decay in relation to the other rates, as shown in Figure 5b. This result may be related to the number of neurons.

3.3 Case 3: Stratified and Intermittent

In training, of the artificial neural network, the Intermittent pattern and the Stratified pattern, the sum mean squared error of the examples is calculated, with equation 1, adding them at the end of each epoch. We analyzed the error behavior during network training in Figure 8.

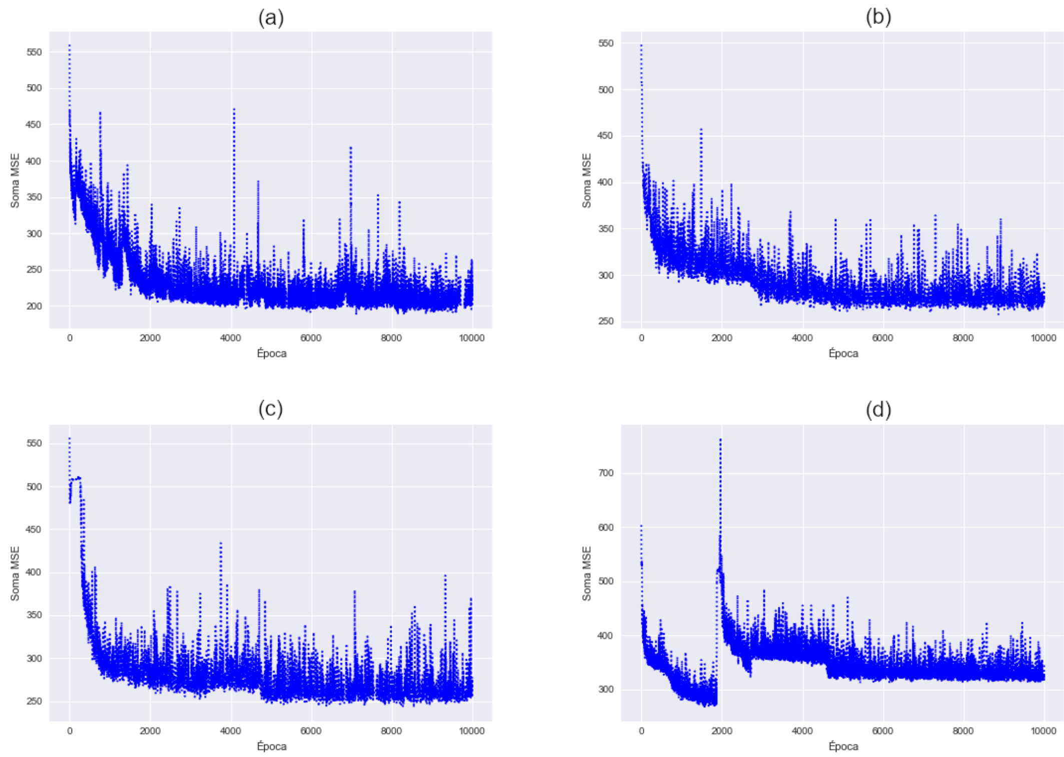

Figure 8 - Sum of the root mean square error of the neural network for Stratified and Intermittent with 25 neurons in hidden layer (a), 50 neurons in hidden layer (b), 75 neurons in hidden layer (c) and 100 neurons in hidden layer (d).

Figure 8 shows behaviors with fast decay going to a constant characteristic after decay. In (a), with 25 neurons, we had the error decay without any interference, (b) also resembles the previous behavior. With 100 neurons, in (d), there is a different behavior, after the initial decay, in 2000 epochs, there is an increase in error, followed by a new decay.

Figure 9 shows the network hit rate during the tests, where new data is presented, different from those shown for the network in training.

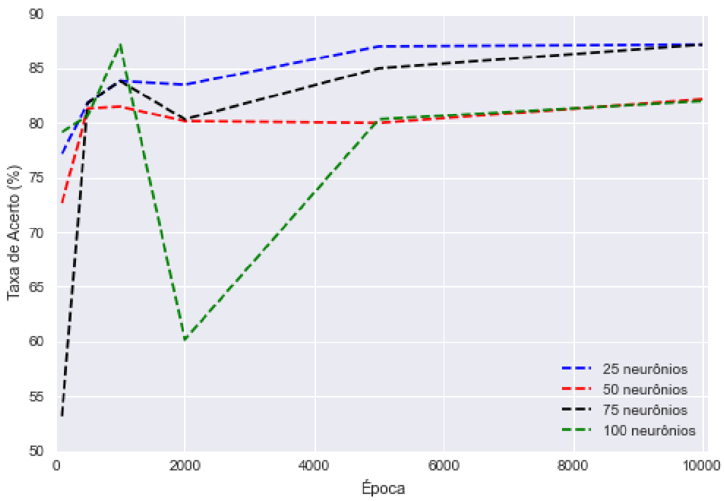

Figure 9 - Stratified and Intermittent hit rate.

The tests show good results for the network hit rate with 87%, with the Intermittent pattern hitting approximately 82% and the Stratified pattern hitting 92%. Associating the error decay behavior in Figure 8a with the hit rate for 25 neurons (blue), it is possible to notice a good performance of the network to recognize the studied patterns. It can be seen that the transition site between the first decay and the second decay phase in Figure 8d directly affected the hit rate for 100 neurons (green). When it reaches the first decay, the hit rate reaches its maximum point in 1000 epochs, with an increase in error in 2000 epochs, the hit rate drops sharply.

3.4 Case 4: Three standards

The 3-pattern network training has 3600 examples with 9 input parameters in each example. The variation of the stopping criterion based on epochs follows the same logic as the simulations with 2 patterns, being 100, 500, 1000, 2000, 5000 and 10000 epochs. The hidden layer neurons range from 25, 50, 75 and 100 neurons. The behavior of training time for the neural network with 3 patterns, time has a linear relationship with the number of epochs in the network stopping criterion. For the configuration of 25 neurons in the hidden layer and 100 epochs, we obtained a time of 4.852 seconds. Increasing the hidden layer neurons to 100 and the stopping criterion to 10,000 epochs, the time is approximately 26 minutes.

The neural network for training 3-pattern prediction has 3 output neurons in the output layer. The measurements of the squared error of each output neuron are added to the other errors of the output layer, after which the mean squared error is calculated for each example presented for the network. At the end of each epoch, when all examples are presented to the network, the sum of all mean square errors of each example is performed, as shown in equation 2. The analysis of this measure is performed for specific points of epochs up to 10000 epochs, showing the behavior of the network in relation to this measure during the different epochs. This analysis takes place for the configuration of 25, 50, 75 and 100 hidden neurons in the network.

$$E = \sum_{n=1}^{3600} \frac{1}{3} \sum_{k=1}^{3} e_k^2 \quad (2)$$

After training the network, the behavior of the error during the simulation is analyzed. Figure 10 shows this behavior with the neuron configurations in the hidden layer.

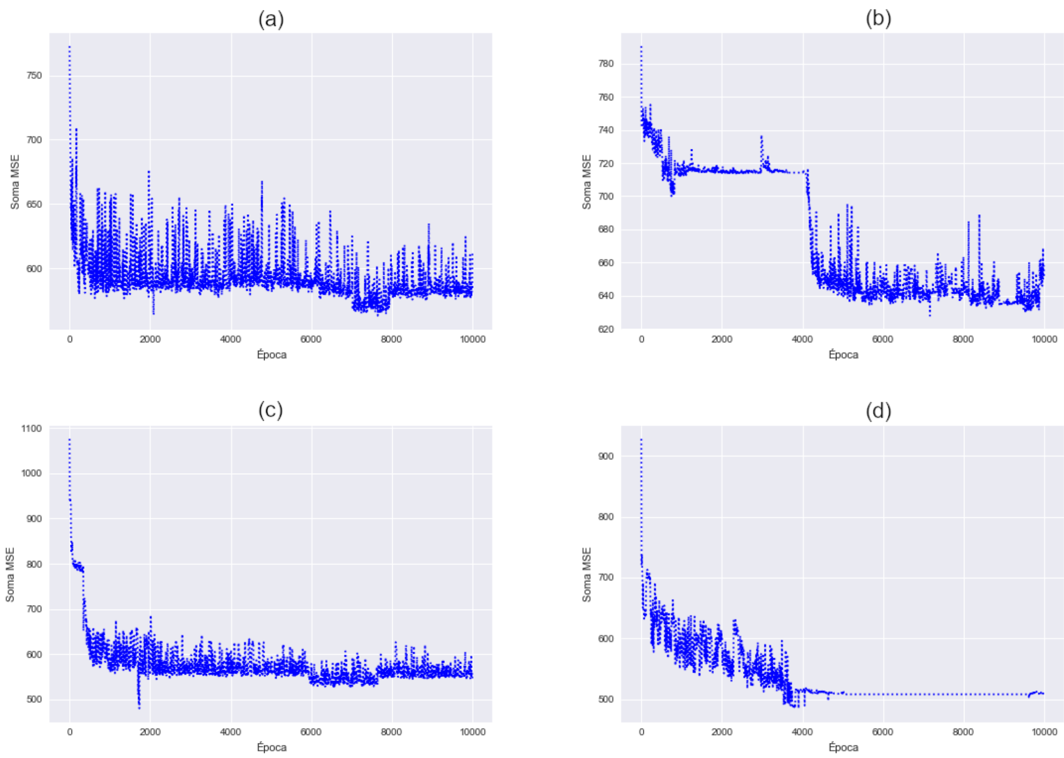

Figure 10 - Sum of the root mean square error of the neural network for 3 patterns with 25 neurons in the hidden layer (a), 50 neurons in the hidden layer (b), 75 neurons in the hidden layer (c) and 100 neurons in the hidden layer (d).

The sum of the error with the 3-pattern neural network shows decay behavior. In (b), for 50 neurons, we noticed two decay parts, where the first is ended with a high error value, then migrating to the second decay from the point where the first part ended. In the first decay, the error value is close to 720, for the second phenomenon, the decay value is around 640.

With the trained neural network, tests are applied with different data from those used during training. We analyzed the hit rate of the test, shown in Figure 11.

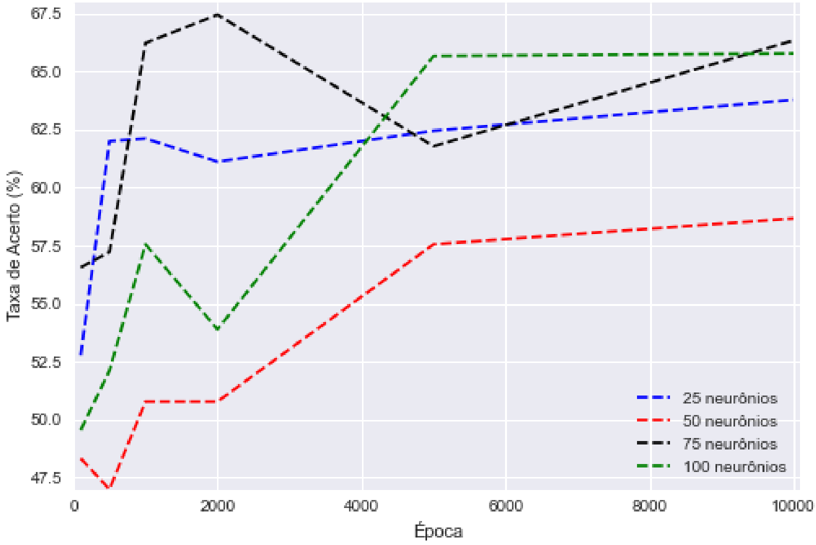

Figure 11 - Hit rate for 3 patterns.

The highest correctness rate in the tests was 67.4%, with approximately 83% correctness of all data in the Stratified pattern, 80% correctness in the Annular pattern and 36% correctness in the Intermittent pattern. The best results in the hit rate were for the configuration of 75 neurons (black) and 100 neurons (green), which presented, in Figure 11, more stable decay behaviors compared to the others. The phenomenon seen in the 50-neuron configuration in Figure 10b was reflected in the performance observed in Figure 11 for 50 neurons. We observed that the hit rate for 100 neurons in the hidden layer is more stable after 5000 epochs, analyzing this region, in Figure 10d, it is noted that there are no oscillations as in the other graphs, reducing these oscillations can make the more stable network in relation to its hit rate.

4. CONCLUSION

There is a lot of complexity to predict the behavior of multiphase flow. Determining these flow patterns during industry operations is of utmost importance. The network results were satisfactory to better understand this tool with the recognition of flow patterns in multiphase flow. Analyzing the behavior of the network during training can clarify several issues that have not been explored yet. We presented as many flow parameters as possible to analyze the behavior of the network with the largest number of observable variables of the phenomenon under study, the parameters were pressure, liquid density, gas density, liquid viscosity, gas viscosity, diameter tube angle, tube angle, liquid surface velocity and gas surface velocity. We did not analyze the blister pattern due to the difference in data between the patterns after treatment.

With the 3 patterns, intermittent, annular and stratified, the network presented a hit rate of 67.4%, with a higher hit rate in the Stratified pattern, followed by the Annular and right after the Intermittent. In this configuration, the Intermittent pattern obtained fewer hits, which can be further explored in the future, but this result may be associated with the difficulty of analyzing the behavior of the gas pockets in the liquid through parameters. When the Stratified and Intermittent patterns were presented, the network showed 87% accuracy, with greater accuracy for the Stratified pattern. Between the Intermittent and Cancel patterns, the net had 90%

correctness, the two patterns have a high correctness rate between them. Finally, the Stratified and Annular patterns presented the worst performance with 68% of correct answers, and the behavior of the net error during training was peculiar.

During training, it is observed that the less oscillatory decay behavior has a better performance in the hit rate. There is still a lot of study to be done to better understand network performance with flow pattern recognition. We will analyze other dimensionless variables for the next works, which can better represent the multiphase flow. Another point to be analyzed is the improvement of the neural network configurations that are most suitable for this phenomenon under study, whether in the activation function, number of hidden layers, number of neurons and stop criterion method.

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