

Velocity distributions in a model of traffic flow

Alexandre M. S. de Melo and Andre M. C. Souza

Departamento de Física, Universidade Federal de Sergipe, 49100-00, São Cristóvão-SE, Brasil

amcsouza@ufs.br

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We use the cellular automaton model in order to describe traffic flow complex variables. More precisely, we study the velocity distributions according to the density of cars. We present the whole spectrum of car velocities using mean-field approximation and numerical simulation. For the maximal high velocity limit of the model both approaches are in very good agreement. However, for maximal intermediate velocity, expected to be qualitatively similar to the experimental data, we have not found much likeness between them.

Keywords: Traffic flow; Cellular automaton.

1. INTRODUCTION

Since the interesting Nagel and Schreckenberg [1] connection between cellular automaton (CA) and relevant highway traffic flow quantities has been proposed [2], much attention has been dedicated, from this standpoint, to this discussion [1-6]. This type of approach has matched well with experimental data.

We consider the CA model constituted of an array of L sites, each site being either empty or occupied by one vehicle at a velocity $v=0,1,\dots,v_{\max}$. We assume a random initial distribution of vehicle positions and null initial velocities. At each time step $t \rightarrow t+1$ the position (S_i) and the velocity (v_i) of the i th vehicle are updated simultaneously (parallel update) according to the following:

- (i) $v_i=v_i+1$ if the distance of the i th car to the next car is greater than v_i+1 , and taking $v_i=v_{\max}$ as a limit.
- (ii) $v_i=D-1$ if the distance to the next car is $D \leq v_i$.
- (iii) $v_i=v_i-1$ with a random probability p if $v_i \geq 1$.
- (iv) The j th car is advanced v_i sites.

If we use a periodic boundary condition (closed circuit), the rules ensure that the total number N of cars is conserved under the dynamics.

The investigation of traffic flow CA is based on the analysis of the fundamental traffic parameters density (ρ) and flux (q). In the stationary state, these results can be compared directly with measurements of real traffic [1]. At low density this model exhibits a laminar flow (free) phase. At high density one expects a start-stop waves (jammed) phase. In this sense, with increasing car density, the notion of a phase transition emerges [5].

2. METHOD AND RESULTS

To look for a possible transition between a free and a jammed phase, we define an order parameter $M = 1 - [q/(v_{\max} \rho)]$. If $p=0$, in the free phase we have $M=0$, and in the jammed phase $M \neq 0$. If $p \neq 0$ the M parameter is not zero anymore, indicating that the probability p is the *conjugated parameter* of the order parameter of a traffic flow CA [7].

Until now, the main interest in the study of the CA model has been the calculation of the so-called fundamental diagram [1-5]. In this paper, we refer to the study of more complex variables through velocity distributions according to the density of vehicles. The whole spectrum of car velocities is motivated by both theoretical interest and various possible experimental investigations on traffic systems.

The analytic solutions of the CA traffic model have been investigated by several authors and only a few exact results has been found. In the low density limit, the flux versus density

solution for v_{max} and ρ is $q=(v_{max} - p)\rho$. In the high density limit, we find $q=(1-p)(1-\rho)$. For $p=0$ we find $q=v_{max}\rho$ if $\rho \leq \rho_c$ and $q=1-p$ if $\rho > \rho_c$, where $\rho_c = 1/(1+v_{max})$. The complete analytical solution for the case of $v_{max}=1$ is given by [6]

$$q = \frac{1 - \sqrt{1 - 4\rho(1-\rho)(1-p)}}{2(1-\rho)} \tag{1}$$

We want to study the velocity distributions of the model. Therefore, we define the probability that there is no car at site i by $h(i,t)$ and car at velocity α ($\alpha = 0, 1, 2, \dots, v_{max}$) at site i and time step t by $c_\alpha(i,t)$. For all sites and all time steps we have

$$h(i,t) + \sum_{\alpha=0}^{v_{max}} c_\alpha(i,t) = 1 \tag{2}$$

$$\rho(i,t) = \sum_{\alpha=0}^{v_{max}} c_\alpha(i,t) \tag{3}$$

and

$$q(i,t) = \sum_{\alpha=0}^{v_{max}} \alpha c_\alpha(i,t) \tag{4}$$

where $\rho(i,t)$ and $q(i,t)$ denote respectively the car density and the flux for site i at time step t .

The first studies on CA traffic flow concentrated on computer simulations and mean-field theories. The mean-field theory neglects spatial correlations using all expected values factorized into local terms [6]. For the stationary state, as the system is homogeneous due to the considered boundary conditions, no parameter will be position dependent. Therefore, according to the update rules of the CA we can find a recursion relation in order to obtain the stationary state solution given by

$$c_0 = \rho^2 \frac{1 + ph}{1 - ph} \tag{5}$$

$$c_1 = (1-p)h\rho^2 \frac{2 - \rho + ph^2}{(1-ph^2)(1-ph^3)} \tag{6}$$

$$c_\alpha = \frac{1 + (1-2p)h^\alpha}{1 - ph^{\alpha+2}} hc_{\alpha-1} - \frac{(1-p)h^\alpha}{1 - ph^{\alpha+2}} hc_{\alpha-2} \tag{7}$$

$$c_{v_{max}-1} = \frac{1 - (1-p)h^{v_{max}}}{1 - (1-p\rho)h^{v_{max}-1}} (1-p)h^{v_{max}-1} c_{v_{max}-2} \tag{8}$$

and

$$c_{v_{max}} = \frac{(1-p)h^{v_{max}}}{1 - (1-p)h^{v_{max}}} c_{v_{max}-1} \tag{9}$$

Therefore, starting with c_0 and c_1 , one can find the mean-field distributions $c_2, c_3, \dots, c_{v_{max}}$ recursively.

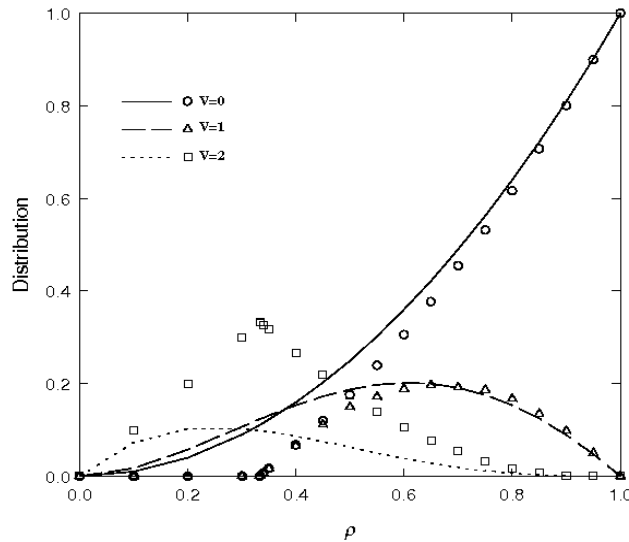


Figura 1: Velocity distributions (C_α) for the mean-field approximation (curves) and for numerical simulation (symbols) with $p=0$ and $v_{max}=2$.

Let us now focus on the numerical treatment. We present the distributions as obtained through simulation in which we have performed about 25 experiments, each running over a time period $T=10000$ after a relaxation time $t_0=10L$, with $L=1000$.

Figures 1 and 2 show velocity distributions versus density curves for $v_{max}=2$ and $v_{max}=4$, respectively, with $p=0$. The symbols indicate the numerical simulation and the curves represent the mean-field approximation (Eq. 5-9). The mean-field (MF) system presents the high velocity ($v_\alpha \approx v_{max}$) distributions going to zero more rapidly than in the simulation. Also, it presents the low velocity distributions going to zero less rapidly than in the simulation. We do not have, consequently, much resemblance between the MF and the simulation system. This fact is clearly related to the uncorrelated cars we have taken in the MF approximation, which makes it difficult to accelerate and stay at high velocities over a certain period of time, thus supporting intermediate velocities. This behavior, with arbitrary values of v_{max} and p have been analyzed, and the corresponding diagrams present the same characteristics. However, when the values of v_{max} increase both approaches are in very good agreement.

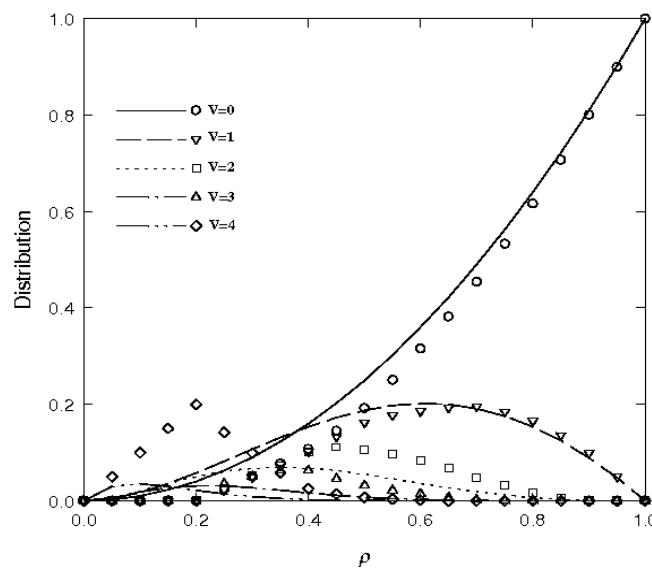


Figura 2: Velocity distributions (C_α) for the mean-field approximation (curves) and for numerical simulation (symbols) with $p=0$ and $v_{max}=4$.

In general, to characterize the velocity distributions of the CA model, we see that for densities close to the lower than critical density (ρ_c) the possibility of low velocities is always quite small as compared to high velocities. If $p=0$ and $\rho < \rho_c$ all cars have a maximal velocity. However, when ρ increases the low velocities distribution increases too.

3.CONCLUSION

To summarize let us say that we have studied the whole spectrum of car velocities of a cellular automaton [1] to describe the traffic flow. This spectrum provides a possible experimental investigation of traffic system, with all car velocities.

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