



# Numerical analysis of ship motion under linear and nonlinear waves

Análise numérica de movimentos de navios sob a ação de ondas lineares e não lineares

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Simulating physical phenomena by applying mathematical and numerical modeling has been increasingly used during the different stages of research and development and innovation projects. The purpose of this study is to present results about translational motion ( $x, y, z$ ), angular motion ( $\alpha, \beta, \gamma$ ) and trajectories obtained using a numerical model developed to study ship dynamics under the influence of linear and nonlinear waves. The results indicated that translational motion, surge and sway, is mostly influenced by thrust in the studied scenario. The translational motion (heave) displayed curves which can be related with wave patterns before 0.1 hr. The angular motion distributions displayed a variation that can be related with wave forces. However, at 0.4 hr the curves depicted smaller variations that is explained by the increased engine thrust pattern. In both simulations, the distance of 20.582 km and 20.342 km for linear and nonlinear simulations were calculated, respectively. Once both simulations yielded similar values for travel distance, the wave patterns used in this study were shown to generate variations at the displacement from 0 to 0.4 hr. After 0.4 hr, the ship displacement depicted linear curves but the engine thrust was able to overcome the external forces.

Palavras-chave: SHIPMOVE, linear waves, nonlinear waves, numerical model

A simulação de fenômenos físicos pela aplicação de modelagem matemática e numérica vem sendo cada vez mais utilizada durante diferentes estágios de pesquisa e desenvolvimento de projetos. O propósito deste estudo é apresentar resultados sobre o movimento de translação ( $x, y, z$ ), rotação ( $\alpha, \beta, \gamma$ ) e trajetórias obtidas usando um modelo numérico desenvolvido para estudar a dinâmica de navios sobre a influência de ondas lineares e não lineares. Os resultados indicaram que os movimentos de translação, avanço e deriva, são muito influenciados pelo empuxo no cenário estudado. O movimento de translação (afundamento) mostrou curvas que podem ser relacionadas com padrões ondulatórios antes de 0,1 h. As distribuições de movimento angular mostraram uma variação que pode ser relacionada com as forças das ondas. No entanto, em 0,4 h as curvas mostraram variações menores, explicadas pelo aumento do empuxo do motor. Nas simulações, as distâncias de 20,582 km a 20,342 km para ondas lineares e não lineares, respectivamente, foram calculadas. Como ambas as simulações mostraram valores similares para a distância navegada, os padrões de onda usados neste estudo mostraram as maiores diferenças de deslocamento entre 0 e 0,4 h de simulação. Após 0,4 h o deslocamento do navio mostrou curvas lineares, mas o empuxo do motor foi capaz de superar as forças externas.

Keywords: SHIPMOVE, ondas lineares, ondas não lineares, modelo numérico

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## 1. INTRODUCTION

The evolution of the shipbuilding industry has resulted in the development of new operating configurations for high speed ships [1]. Reasons for new operating configurations include: ocean articulated convoys, phase approach of the side discharge for tankers, and supplying fuel to military vessels.

Based on studies of ship movement under the influence of waves, many theories have been developed for vessel dynamics, the first study was conducted by Froude (1862) [2]. In his study of ship movements, Froude assumed that the large angles of movement were the result of a successive set of waves, rather than a single wave. However, this author did not investigated perturbations produced by the ship on the flow.

Haskind (1946) [3] studied the sinking and pitching movements of vessels in waves using Green's theorem to calculate the velocity potential associated with oscillatory movements. Haskind was the first to propose the separation of the non-permanent velocity potential into radiation contributions, diffraction and the incidence of waves. However, some fundamental problems inherent in the Thin Ship theory limited the applicability of the method, including the fact that real ships are not thin but are slender, containing draughts of the same order of magnitude as the breadth but not the length.

The sea keeping performances of two model ships in regular waves were studied by Cha and Wan (2015) [4] using their house solver model. Ship motions were accurately predicted in order to study the sea-keeping performances. A comparison of the influence of different hull forms was chosen as the main task. In this study, the heave and pitch were simulated, and the green water phenomenon happened during the ship motion.

Over the years, several methods have been developed in order to investigate the behavior of ships under the influence of external forces. Thus, the purpose of this study is to present results about translational and angular motion, and trajectories obtained using a numerical model developed to study ship dynamics under the influence of linear and nonlinear waves.

## 2. METHODOLOGY

The SHIPMOVE (SHIP MOVEMENT MODEL - BR512014001163-1) model has been developed at the Universidade Federal do Rio Grande to describe ship movement in the time domain using variational theory and Lagrangian mechanics.

### 2.1. Mathematical model

The model considers the effects of waves and the water density field on the behavior and displacement of vessel and accounts for inertial forces, damping and additional masses. The ship can move in three directions under six degrees of freedom, it can perform translational ( $x, y, z$ ) and rotational ( $\alpha, \beta, \gamma$ ) movements. The translation of the  $x$ -axis is called surge, the translation of the  $y$ -axis is the sway and the translation in the  $z$ -axis is called heave. The rotation  $\alpha$  is called roll, the rotation  $\beta$  is the pitch, and the rotation  $\gamma$  is called yaw [5].

The system of ordinary differential equations that represents the dynamics of the vessel with six degrees of freedom ( $x, y, z, \alpha, \beta, \gamma$ ) is represented by Eq. (1).

$$(I + M_A)\ddot{\sigma} + B\dot{\sigma} + F_c(\alpha, \dot{\alpha}) = T_m + F_e \quad (1)$$

where  $I$  represents the inertia matrix,  $M_A$  is the additional mass matrix,  $B$  represents the potential damping matrix, and  $F_c$  represents the centrifugal efforts of the system. The external forces are represented by  $F_e$ , and the thrust of engine is represented by the  $T_m$  matrix.

### 2.2. External Forces

In this study, a variation of the Morison equation (Eq. 2) was used to represent the external forces on the numerical model. The Morison Equation considers the fluid acting on an elliptical geometric structure. It is often used to describe the effect of horizontal hydrodynamic forces generated by progressive waves on slim bodies. This equation can be decomposed into the sum of the drag and inertial forces [6].

The equation can be decomposed into three terms. The first term represents the drag force between the ship and the water. The second term represents the acceleration and deceleration of the ship, which generates a pressure gradient (this term is known as the Froude-Krylov force). The third term refers to the inertial forces, which depend on the relative acceleration between the vessel and the water [7].

$$F_{ext} = \frac{1}{2} \rho C_D D |v_n - u_n| (v_n - u_n) - \rho A \dot{u}_n h (C_M - 1) + \rho A \dot{v}_n C_M \quad (2)$$

where  $\rho$  is the density of the fluid,  $C_D$  is the drag coefficient,  $C_M$  is the inertia coefficient,  $D$  is the transverse dimension of the body,  $v_n$  and  $\dot{v}_n$  are the velocity and acceleration of the fluid, respectively, and  $u_n$  and  $\dot{u}_n$  are the velocity and acceleration of the vessel, respectively.

To calculate the density of the fluid, the international equation for the state of seawater, which used real data on salinity and temperature obtained in the region of the Patos Lagoon estuarine channel, was used. A standard atmosphere with zero pressure was used to perform the calculations [8].

The influence of the waves was included through the use of the significant wave height, acceleration and velocity of the wave components. In this study linear and nonlinear waves were used (Fig. 1). Airy wave theory describes the linear propagation of gravity waves defining linear waves by sine or cosine function [9]. Stokes wave theory describes nonlinear propagation of gravity waves. The nonlinear gravity waves can be generalized by linear small-amplitude harmonic waves according to Stokes (1847) [10]. The maximum significant height was 0.7 m and the maximum velocity was 0.65 m/s for linear waves. For nonlinear waves the maximum significant height was 1.65 m and maximum velocity was 1.57 m/s.

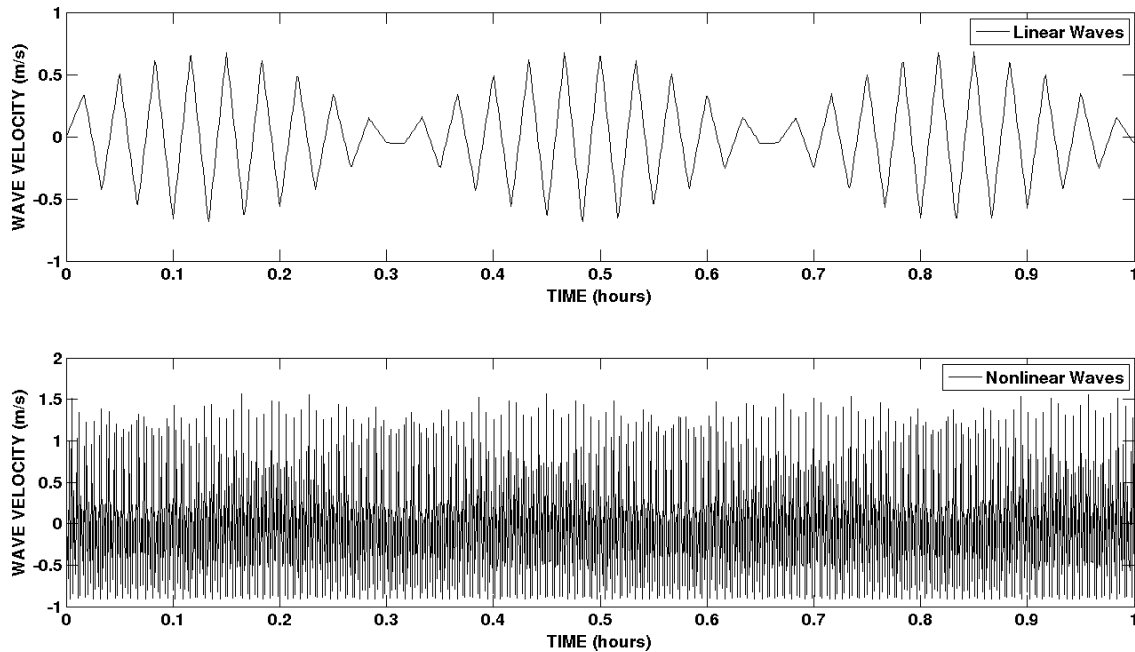


Figure 1: Wave velocity for linear and nonlinear waves, respectively.

### 2.3. Thrust

The thrust applied to the vessel was directly related to the resistance force from the displacement of the vessel on the fluid at a controlled speed [11]. The ship is initially at rest and then it assumes a respective thrust imposed by the ship engines (Fig. 2). Since the performance of the two simulations used the same ship, both analysis have the same thrust curve (Fig. 2). At 0.01 hr the thrust start to display values different than 0 N. At 0.4 hr the thrust is  $1.73 \times 10^6$  N, force value able to overcome wave forces. The highest thrust value is  $11.21 \times 10^6$  N.

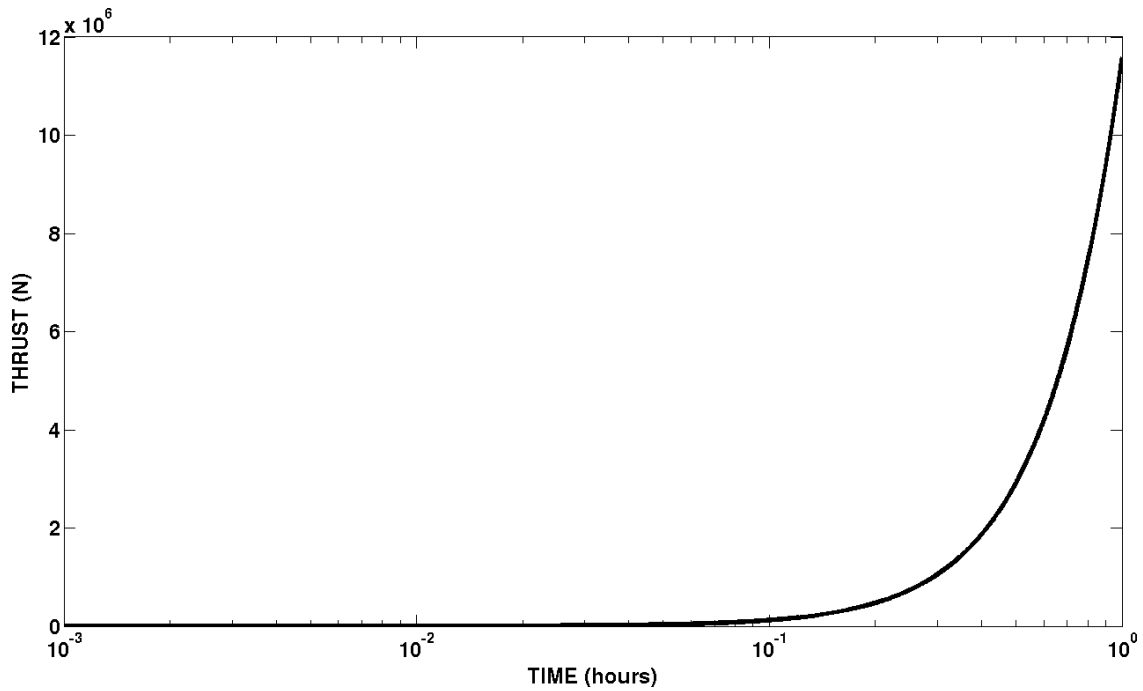


Figure 2: Thrust applied to the vessel, simulations with linear and nonlinear wave patterns.

### 3. RESULTS ANALYSIS

In order to investigate the behavior of the ship, the simulations were performed by 1 hour which used a ship with a length of 138 m, a width of 24 m and a gross tonnage of 11651 ton regarding two different wave events, linear and nonlinear wave patterns (Fig. 1). The translational motions, surge and sway, managed to depict similar results for both simulations (Fig. 3). It is possible to observe that surge and sway are influenced by the thrust. The surge motion also showed greater values at 0.01 hr, such as the thrust, reaching the position of 18608.0 m. The sway displayed bigger values at 0.025 hr reaching 8794.1 m. The heave distribution depicted a small contrast between both simulations, however both yielded small values. At 0.1 hr, the heave displayed a regular pattern that can be related with greater values of thrust.

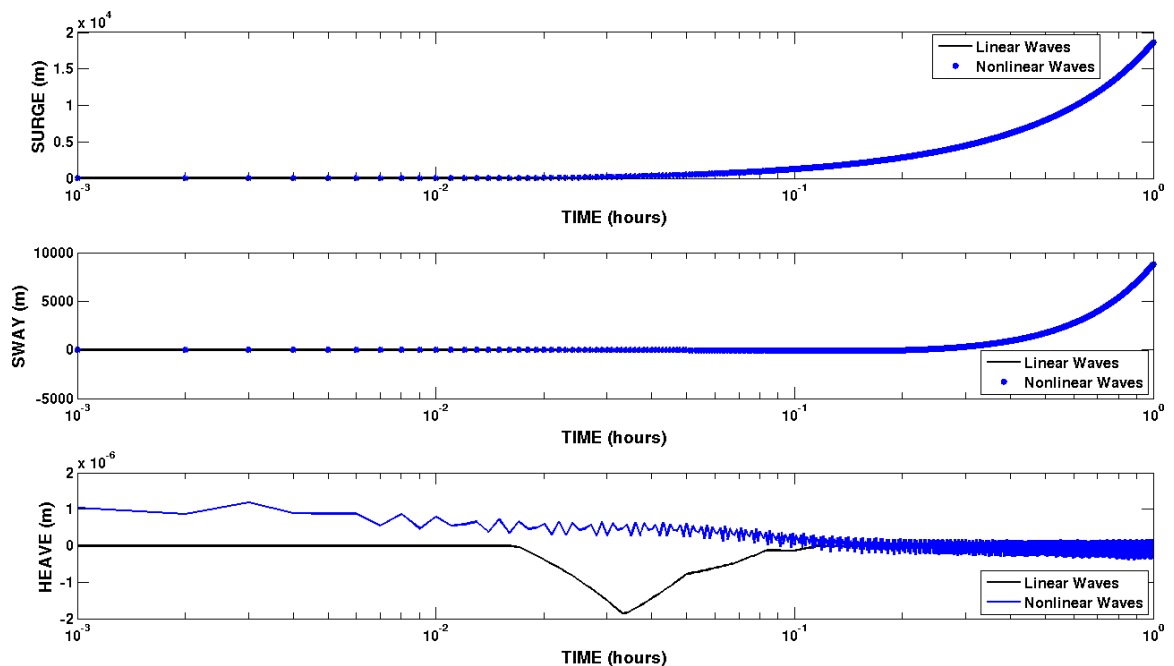


Figure 3: Translational motion for simulations with linear and nonlinear wave patterns, respectively.

The simulation with linear waves generated a small variation on the pitch distribution where the maximum angle was  $-0.34^\circ$  (Fig. 4); it can be associated with the thrust which increased at 0.4 hr. For the nonlinear wave simulation, the pitch displayed a variation between 0.001 and 0.2 hr. This behavior is explained by the wave forces that are able to change the displacement of the vessel. Around 0.4 hr the values of pitch increased reaching an angular value of  $-0.51^\circ$ . At the same time, 0.4 hr, the thrust also increased. In other words, the greater pitch angles can be explained by the thrust pattern.

In Fig. 4, the roll motion demonstrated the same pattern for both simulations, however the nonlinear waves depicted greater angular variations than the linear waves,  $0.34^\circ$  and  $0.32^\circ$ , respectively. It is possible to observe, at the end of both simulations, that the two curves present smaller differences. It can be associated with the engine thrust which allows the vessel to overcome the wave force.

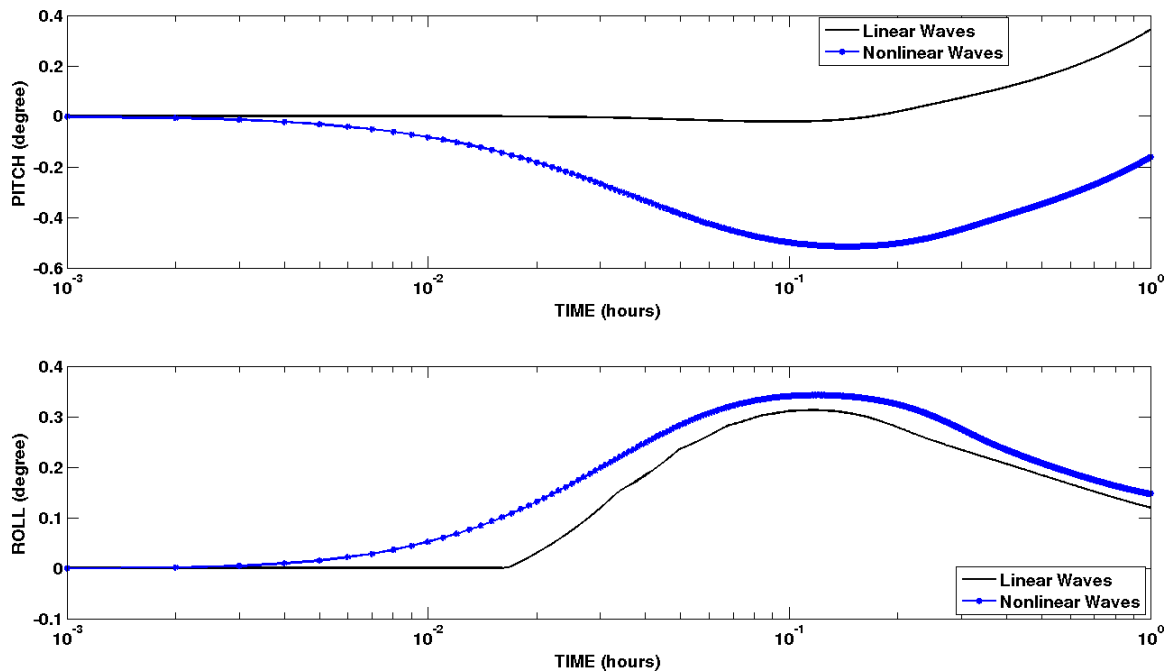


Figure 4: Angular motions, pitch and roll, for simulations with linear and nonlinear wave patterns, respectively.

The yaw motion distribution showed a distinct pattern among the two simulations (Fig. 5). At the start of the linear wave simulation, the yaw curve depicted values relatively close to zero. Within about 0.01 hr, the yaw values increase, attaining an angle of  $0.34^\circ$  and yields a variation pattern that can be associated with the wave pattern. For the simulation with nonlinear waves, the yaw motion displayed little variations that can be related with the nonlinear waves. The maximum angle was  $0.38^\circ$ . For both distributions, the wave effect was expressive. However, at 0.4 hr the curves depicted smaller variations that are explained by the thrust. When the thrust increased the vessel was able to overcome the wave forces. Then the yaw motion curve displayed smaller variations.

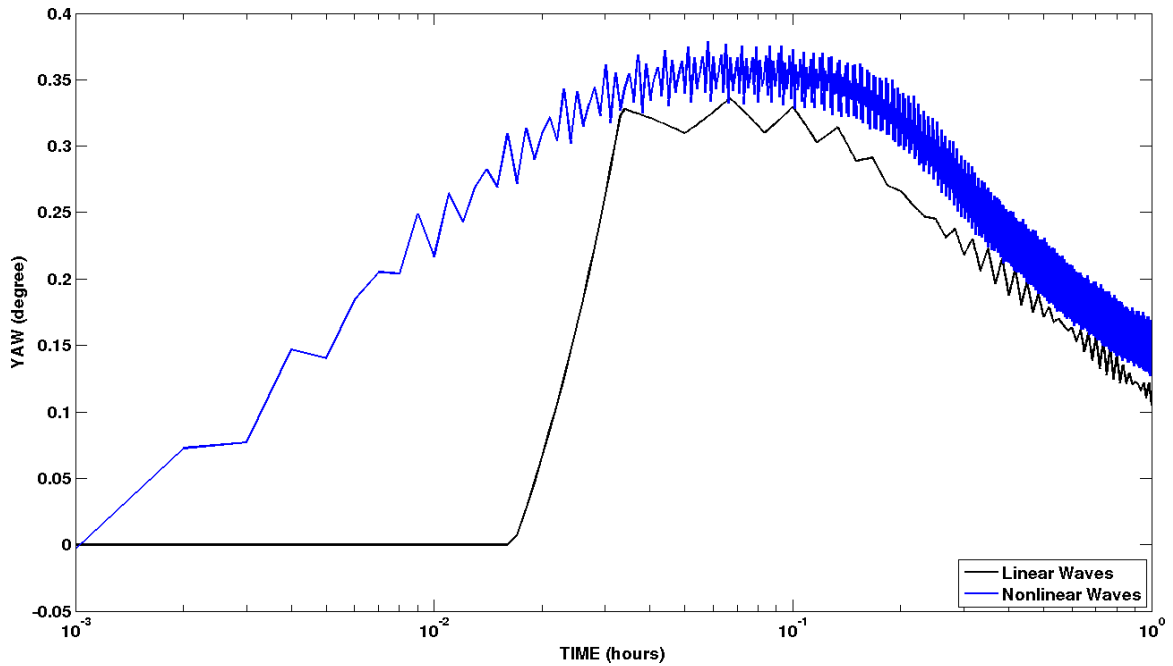


Figure 5: Angular motion, yaw, for simulations with linear and nonlinear wave patterns.

The distance the ship traveled in one hour is shown for both simulations in Fig. 6. At the initial points of trajectories, between 0 and 0.4 hr, both curves depicted variations that can be related to the wave forces. The curves then yield a linear pattern that can be explained by the engine thrust. At 0.1 hr the engine thrusters show an increase in values. Therefore, the vessel was able to overcome the wave forces that created a linear displacement. The mean velocity was 11.12 knots and 10.98 knots, for the simulated vessel with linear and nonlinear waves, respectively. The total traveled distance of the ships were 20.582 km and 20.342 km for the simulations with linear and nonlinear waves.

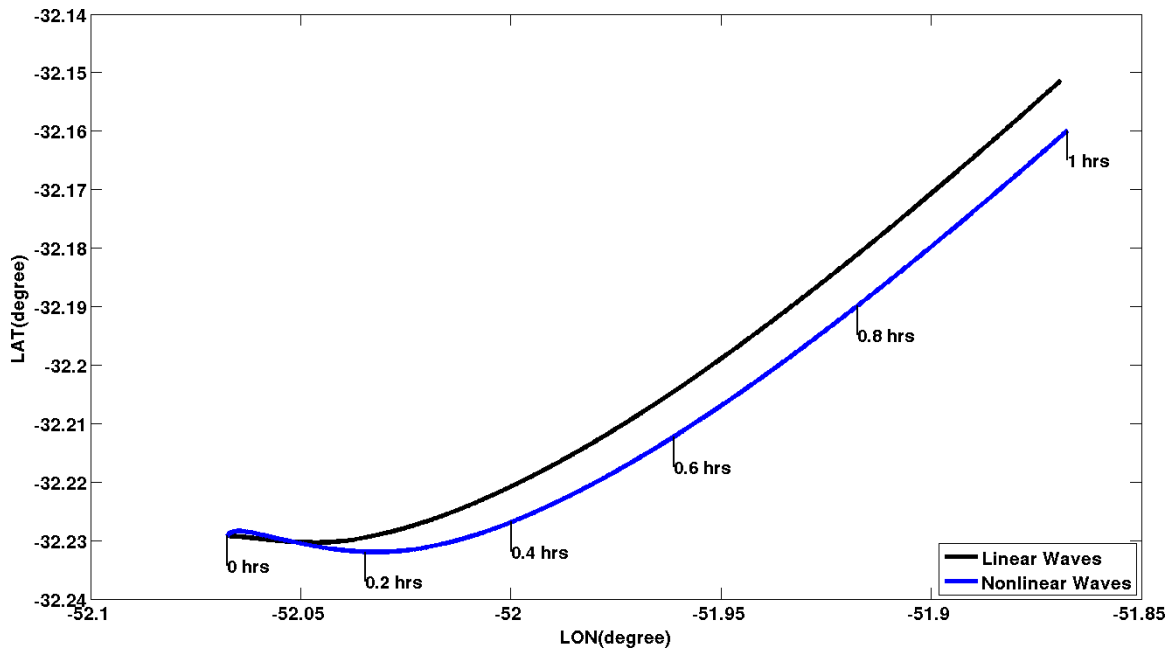


Figure 6: Ship displacement under linear and nonlinear waves.

Certain papers promote similar analysis. Cha and Wan (2015) [4] developed a numerical investigation of motion response for two model ships in regular waves. The results for the pitch were regular sinusoidal or co-sinusoidal curves, in which depicted linear characteristics. The regularity of the heave was just the same as the pitch at different conditions. The numerical results showed that the motion fluctuations in the vertical direction or about the  $y$  axle present typical linearity, and nonlinearity is aggravated due to the increase of the hull velocity and wave height.

In the present study regularity among the heave and pitch was not found. Surge ( $x$  axle) and sway ( $y$  axle) motions presented a typical linearity in the first 0.01 hr of simulation that was displayed in Fig. 3. However the vertical direction ( $z$  axle) or heave motion did not show linearity for the simulation with nonlinear waves. For the simulation with linear waves, heave motion displayed linear results until approximately 0.005 hr.

An investigation of head-sea parametric rolling and its influence on container lashing systems was developed by France et al. (2003) [12]. This paper provides insight into the conditions in which post-Panamax container-ships were likely to experience head sea parametric rolling, and the magnitude of motions and accelerations that can occur. They also discussed how such extreme motions impact the design and application of container securing systems.

The model tests were performed on the post-Panamax C11 container-ship constructed in the new Sea-keeping and Manoeuvring Basin (SMB). The main purpose of the model tests was to understand vessel motions during the storm encounter when extensive loss and damage of containers occurred. During the simulation on 8 m regular waves, the model was pitching to angles of about  $4^\circ$ , with negligible roll response. A small excitation, likely introduced by a rudder movement, causes the vessel to take a small roll to one side. Roll angles then increased from a few degrees to over  $30^\circ$ .

In the present study, for linear or regular waves, the maximum pitch angle was  $0.34^\circ$  and the maximum roll angle was  $0.30^\circ$ . The big difference between this study and that of France et al. (2003) [12] can be explained by the wave height. The maximum significant wave height that was used in the present study was 0.7 m, while France et al. (2003) [12] simulated their model using a height of 8 m. Analysing both results, it is possible to say that the height had a significant influence on the angular motions.

A mathematical modeling of a ship motion in waves under coupled motions was developed by Thu et al. (2015) [13]. In this paper, the research area is emphasized on motion of a ship due to anti-symmetric coupled motion of roll-yaw and sway-roll-yaw. The governing equations of motion are solved numerically using Runge-Kutta method. The mathematical model presented is based on the drawing lines and load conditions.

A coupled roll-yaw mathematical model as followed based on a nonlinear strip theory is used to calculate yaw and roll motions in regular head seas with parametric rolling taken into account. Incident waves are unidirectional and of single periodicity. The system with six degrees of freedom moving ship in waves can be considered as a linear mass-damping-spring system with frequency dependent coefficients and linear exciting wave forces and moments.

In order to simulate the motion of ship required frequency dependent hydrodynamic coefficients, related to sectional added mass, damping and wave exciting force are adopted from the experimental results. For the results simulated with wave frequency of 0.56 Hz, the roll motion depicted values between  $-0.1^\circ$  and  $0.1^\circ$ . Yaw motion displayed angles between  $0^\circ$  and  $-0.1^\circ$ , approximately. When the wave frequency increased the roll and yaw angles decreased. The maximum sway displacement for this frequency was 10 m. It also decreased when the angle increased.

The angular motions showed in the present study depicted the same order of magnitude displayed on the article by Thu et al. (2015) [13]. The translational motion sway depicted greater values when compared with that study. The maximum sway value was 8700 m. Thu et al. (2015) [13] simulated only 120 s, while this study simulated 1 hr or 3600 s. In this present study, the sway motion at 120 s displayed 11.7 m. So, both models seem to show similar results for angular and sway motions. This similarity can be explained by the mathematical formulation. Both models used the system of ordinary differential equations to represent the dynamics of the vessel with six degrees of freedom (Eq. 1).

The next step forward is to carry on simulations of SHIPMOVE coupled to a numerical system TELEMAC ([www.opentelemac.org](http://www.opentelemac.org)) using three-dimensional hydrodynamics and wind wave modules. This coupled system will be used to force the boundary conditions and consider more realistic scenarios of ship manoeuvring at the adjacent coastal region of the Patos Lagoon.

#### 4. CONCLUSION

The SHIPMOVE was simulated with linear and nonlinear waves. The results showed that the ship behavior is mainly influenced by the thrust and wave forces. In both scenarios the translational motions, surge and sway, depicted the same distribution for both simulations. These distributions presented the same pattern, so it is possible to conclude that translational motion, surge and sway, is mostly influenced by thrust in the studied scenario. On the other hand, the translational motion (heave) displayed curves which can be related with wave patterns. However, at 0.1 hr, the heave depicted a regular pattern that can be related with greater values of thrust.

The angular motion distribution (pitch) depicted a pattern that can be related with the engine thrust for both simulations. However, for the nonlinear wave simulation the pitch displayed a variation that can be related with wave forces. The yaw curves showed a pattern influenced by the wave forces, for both simulations. However, at 0.4 hr the curves depicted smaller variations that are explained by the increased engine thrust pattern.

Both simulations promoted a significant travel distance, 20.582 km and 20.342 km for linear and nonlinear simulations, respectively. Once both simulations yielded similar values for travel distance, the wave patterns used in this study were shown to generate variations at the displacement (0 - 0.4 hr). After 0.4 hr, the ship displacement depicted linear curves. Such behavior is explained by the engine thrust that was able to overcome the external forces. So, the vessel was able to defeat the wave forces, and promote a complete linear pattern of displacement.

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